A Note on Lykken's Expected Ratio of Black-to-White Crime Rates

Lykken (1995), a landmark study on antisocial behavior, emphasizes grave negative effects of illegitimacy on crime. Indeed the much higher Black crime rates compared to White ones coexist with the much higher Black single parenthood.

With the definitions of

P(·|·): conditional probability

C: crime rate

B: being Black

W: being White

1: being raised under a single parent

2: being raised under both parents,

The ratio of Black-to-White crime rates *LR* can be expressed as

$$LR = \frac{P(C|B)}{P(C|W)} = \frac{P(C \cap 1|B) + P(C \cap 2|B)}{P(C \cap 1|W) + P(C \cap 2|W)}$$
$$= \frac{P(C|1 \cap B) \cdot P(1|B) + P(C|2 \cap B) \cdot P(2|B)}{P(C|1 \cap W) \cdot P(1|W) + P(C|2 \cap W) \cdot P(2|W)}.$$

On the other hand, Lykken defines his expected ratio of Black-to-White crime rates *RC* as[1]

$$RC = \frac{P(C|1) \cdot P(1|B) + P(C|2) \cdot P(2|B)}{P(C|1) \cdot P(1|W) + P(C|2) \cdot P(2|W)}.$$

Unless both of

$$P(C|1 \cap B) = P(C|1 \cap W) = P(C|1)$$
$$P(C|2 \cap B) = P(C|2 \cap W) = P(C|2)$$

hold, *LR* does not equal *RC*. As pointed out by Lykken himself, *LR* is larger than *RC* or

$$\frac{LR}{RC} = \frac{\dfrac{P(C|1 \cap B) \cdot P(1|B) + P(C|2 \cap B) \cdot P(2|B)}{P(C|1 \cap W) \cdot P(1|W) + P(C|2 \cap W) \cdot P(2|W)}}{\dfrac{P(C|1) \cdot P(1|B) + P(C|2) \cdot P(2|B)}{P(C|1) \cdot P(1|W) + P(C|2) \cdot P(2|W)}} > 1.$$

This inequality can happen either because

$P(C|1 \cap B) > P(C|1 \cap W)$, $P(C|2 \cap B) > P(C|2 \cap W)$, or both.

Although his conjectures on causes of the *LR*-*RC* inequality (pp. 216-218) are quite plausible, he is not explicit about the fact that his RC formula is based on the

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[1] Lykken (1995, p. 215). The equivalence of his definition and my expression hereof is shown in Appendix.

assumptions of

$$P(C|1 \cap B) = P(C|1 \cap W) = P(C|1)$$
$$P(C|2 \cap B) = P(C|2 \cap W) = P(C|2).$$

Consequently, a very interesting question, that is, the Black-to-White crime rates difference between being raised under a single parent and both ones, is not explicitly considered.

Appendix

Lykken defines the expected ratio of Black-to-White crime rates as

$$RC = R \times \frac{(S-1) \cdot P(1|B) + 1}{(S-1) \cdot P(1|B) + R}$$

with the definitions of

$$R = \frac{P(1|B)}{P(1|W)}, \ S = \frac{P(C|1)}{P(C|2)}.$$

Inserting these definitions into the expected ratio formula, we obtain

$$RC = \frac{P(1|B)}{P(1|W)} \times \frac{(S-1) \cdot P(1|B) + 1}{(S-1) \cdot P(1|B) + \frac{P(1|B)}{P(1|W)}}$$
$$= \frac{P(1|B)}{P(1|W)} \times \frac{(S-1) \cdot P(1|B) + 1}{(S-1) \cdot P(1|B) + \frac{P(1|B)}{P(1|W)}} = \frac{(S-1) \cdot P(1|B) + 1}{(S-1) \cdot P(1|W) + 1}$$
$$= \frac{(S-1) \cdot P(1|B) + P(1|B) + P(2|B)}{(S-1) \cdot P(1|W) + P(1|W) + P(2|W)} = \frac{S \cdot P(1|B) + P(2|B)}{S \cdot P(1|W) + P(2|W)}$$
$$= \frac{\frac{P(C|1)}{P(C|2)} \cdot P(1|B) + P(2|B)}{\frac{P(C|1)}{P(C|2)} \cdot P(1|W) + P(2|W)}$$
$$= \frac{P(C|1) \cdot P(1|B) + P(C|2) \cdot P(2|B)}{P(C|1) \cdot P(1|W) + P(C|2) \cdot P(2|W)}.$$

Reference

Lykken, D. T. 1995. *The Antisocial Personalities*. New York, U.S.A.: Psychological Press.